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A REVIEW ON VARIOUS BEAM THEORY TECHNIQUES FOR DYNAMIC ANALYSIS OF ROTATING LAMINATED SHAFTS

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ABSTRACT

The composite laminated shafts have wide ranging applications in various mechanical devices that include cylindrical tubes used in the marine and automotive industries and centrifugal separators of helicopters. Rotating laminated composite shafts are developed for the replacement of traditional metallic shafts. An efficient beam theory technique is required to analyse the changing behaviour of the rotating laminated composites. In recent times many of the beam theory techniques that have been proposed analysed the dynamics of the laminated shafts that are rotating in nature. The present paper presents a comprehensive review based on beam theory techniques that include Equivalent Modulus Beam Theory (EMBT), Simplified Homogenized Beam Theory (SHBT), Modified EMBT, Layer wise Beam Theory (LBT), Timoshenko beam theory (TMBT), Euler-Bernoulli beam theory and Shear Deformation Beam Theory (SDBT). From a review of these techniques, Layer wise Beam Theory (LBT), Timoshenko beam theory (TMBT) and Shear Deformation Beam Theory (SDBT) techniques have been proved to provide better performance than other beam theory techniques in terms of critical speed, accuracy, natural frequencies, and stiffness. Moreover, this paper makes a comparative analysis based on the technique used and their corresponding critical speeds.

KEYWORDS: Layer wise Beam Theory (LBT), Timoshenko BT (TMBT) & Shear Deformation BT (SDBT)

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Original Article

INTRODUCTION

In recent years, the composite laminated shafts are used in several mechanical devices such as: (Ghoneam et al., 2011; Nezhad et al., 2017; Zorzi& Giordano, 1985). Composite material shafts developed to replace the traditional metallic shafts have wide ranging usage and applications . Composite shafts have the various essential characteristics like high strength compared to weight than metals which are traditionally enticing for use in rotating systems and their applications (Banerjee et al., 2016; Yongsheng et al., 2013). When compared to the metallic shaft, the composite laminated shaft has lower level of vibration, better ratio of the strength and weight and a long service life. Also, these materials enable the designers the chance of gaining pre-decided performances such as the attainment of critical speeds, by altercation of the diverse composite layers including the number and orientation of layers (Fanmaleki, &Qatu, 2013; Jacquet-Richardet et al., 2011). These above mentioned advantages of composite material shafts have led to several studies concentrating on the analysis of these shafts and their applications. These studies have produced multiple dissimilar analysis of the dynamic performance of the composite laminated shafts (Chang et al., 2004; Hosseini et al., 2009.

For rotating composite laminated shafts, internal damping is the most significant and a related to a metal rotor (Singh & Gupta, 1996). A driveshaft of the tail rotor which is made of boron (or epoxy) was examined for the rotating materials used in helicopter. (Zinberg and Symonds, 1970). The examination revealed the change in

shear-normal coupling and the coupling effects of bending and stretching with variation in stacking sequence, and modify the frequency valuation . EMBT was used to estimate the critical speed by considering the shaft as a narrow edged circular tube maintained at each ends (Dos Reis et al., 1987; Gupta & Singh, 1996; Jin et al., 2016; Montagnier & Hochard, 2013; suna, 2014). The speed of the shaft was evaluated through estimation of the cure that represented the unbalanced results obtained at the sub-critical zone. A model was developed based upon the methodical examinations of narrow layered composite cylindrical tubes (Chatelet et al., 2000). Layer wise Beam Theory (LBT) was developed by considering a layer-wise displacement field. The theory was also amplified to analyse the dynamic issues of composite rotor (Singh & Gupta, 1996; Qatu& Iqbal, 2010; Shahrjerdi&Yavari, 2018).

A framework has been proposed by Bert, (1993) for diagnosis of critical speeds and the consequences of bending-twisting coupling of the composite shaft. In the stability analysis of the composite shaft the parameters considered are finite elements and loads compressed axially. Timoshenko beam theory is used to design the laminated composite shaft for this analysis (Chen &Peng, 1998; Rajasekaran. 2013). The shaft was designed as a Bresse-Timoshenko beam as Bresse-Timoshenko theory includes shaft gyroscopics (Chang et al., 2004; Elishakoff et al., 2009). The variability nature of composite drive shaft due to fluctuation in rotational speed and torque was studied by Kim & Bert (1993). Multiple narrow shell theories were used by them to conduct the study. Coriolis and centrifugal forces were part of rotational effects. The theory also referred dynamic instability areas for shafts through simple support while longer duration. Vibration characteristics of the rotating composite shafts was studied by Chang et al., (2004). The model developed from that study included gyroscopic effects, rotary inertia and the effects of coupling which were produce as a result of laminating the composite layers. The shear deformation beam theories of first as well as higher orders were used to examine the frequency analysis of FG beams having variable thresholds (Shufrin & Eisenberger, 2005; Simsek, 2010). There are various researches presented the composite rotating shaft model using a first-order shear deformable beam theory (Kant & Gupta, 1988; Reddy, 1997; Zhu et al., 2012; Hajianmaleki & Qatu, 2012).

LITERATURE REVIEW

Various techniques were implemented for modelling of composite laminated shaft, with the aim of dynamic analysis of composite rotors (Gupta, 2015). These techniques include EMBT, SHBT, Modified EMBT, LBT, TMBT, Euler-Bernoulli beam theory and SDBT. Figure 1 shows that the various beam theory techniques used for composite laminated shafts.

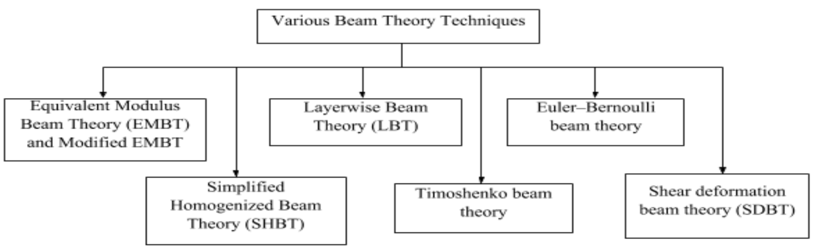

Figure 1: Various Beam Theory Techniques

EMBT & Modified EMBT

Zinberg & Symonds utilised EMBT for the study of critical speeds of anisotropic cylindrical shafts which rotated. The study developed an analysis procedure for the composite shaft. The dynamical evaluation procedure was meant to be

carried out on the narrow edged shaft having internal damping. EMBT theory proposed by Gupta & Singh, (1996) analysed the tubular composite shafts. The shafts were symmetric in structure. The analytical theory was also applicable to dynamic rotor. However, the model based upon EMBT was found to be unsuitable for unsymmetric structures. Application of the theory to all models of dynamic rotor can result in faulty performance prediction (Gayen, 2013). Study by Sino et al., (2008) focused on the rotating composite laminated shaft and its dynamic instability characteristics. The study proposed an integrated finite element beam model while taking internal damping into account. This model measured the instability threshold and natural frequencies. The study also evaluated the relationship between instability threshold and natural frequencies of the shaft with the laminate matrics. The components of laminate matrics were fiber orientation, stacking sequences, and transversal shear effect. It was found that laminate matrics had considerable impact on the instability threshold of the shaft. The characteristics of stability for the composite shaft which is rotating in nature was studied by Chen et al., (1997). The study scrutinised the shaft behaviour in relation to axially compressed load while considering the finite element technique. EMBT was used to construct a Timoshenko shaft from the sample composite shaft with laminations. The results were then compared with the other beam techniques. It was found that the stacking sequence, the boundary conditions and the length-radius proportion affect the critical speed of the composite shaft having thin walls. The study also examined the relationship between location of the disk and the rotational speed depending on stability of the rotating shaft. Traditional buckling formulation was employed to examine the critical load bearing capacity of the rotating shaft which has thin walls. Natural frequencies for composite tubular shafts were examined by Gubran& Gupta, (2005). The study altered gyroscopic effects, EMBT with shear deformation, and rotary inertia for conduction of research. The implications of stacking sequences on the shaft were considered for the study. As the end result of the study, various coupling procedures were proposed by using composite materials. The study shows that the proposed modified EMBT provides more simplicity and more efficient for dynamic rotor analysis of tubular composite shafts.

SHBT

Sino et al., (2006) investigated the dynamic instability analysis of both internal and external damping using general homogenized beam theory. In this, natural damping is evaluated from the specific damping measurements of each laminate layer. This method provides the benefit of being effective for any configuration of the composite wall. developed SHBT was proposed by Sino et al., (2008) for analysing the rotating shaft which is damped internally. This technique overcomes one of the major issues in EMBT. In EMBT, the symmetrical and balanced stacking sequences are incorporated, without any considerations for the distance of composite layers from neutral axis. In this, the critical speed attained by the algorithms is better than the EMBT and LBT in terms of instability thresholds.

A first and second-order Generalised Beam Theory (GBT) was proposed by Davies, (1998). The theory was meant to solve the issues arising out of distortional buckling of cold-formed sections. The theory used finite element modelling to get the desired results. Nevertheless, GBT will always provide the most sophisticated output. The segregation of essential modes, in GBT also enables opportunities that are not available in other alternate approaches. Silvestre, (2007) investigated and analysed the elastic buckling characteristics of Circular Hallow Section (CHS) using a generalized beam theory. In this, the beam theory technique is used for adaptation by considering the specific criteria like cross-section geometry. Also, in this theory, the kinematic relations of thin shells and the variance of strain energy are analysed as they constituted the structural characteristics of the Circular Hallow Section. In addition, the study also examined a cluster of shell-type deformation modes and modelling consists of torsion and axisymmetric deformation modes.

Layer Wise Beam Theory (LBT)

For examination and accurate estimation of delamination and interlaminar stress dispensation, layerwise laminate theories are used. In case of anisotropic materials, the chances of reduction in transverse shear effects are negligible, and hence in these cases the above mentioned theories are utilised. In a study by Singh and Gupta, (1996), EMBT and LBT were analysed. The study produced mathematical model so as to analyse working procedure of composite rotor. A layer wise theory was proposed by Lee, (2000) &Gubran, (2000) to examine free vibration. The theory evaluated delamination based laminated beam. Sekhar&Srinivas, (2002) proposed vibration characteristics of slotted composite shafts using beam theory techniques. Yongsheng et al., (2014) proposed a rotating thin-walled composite shaft using layerwise beam theory with internal damping.

On dimensional layer-wise theory was suggested by Filippi& Carrera, (2016). For this theory contrived layers of cross-sectional layers and higher-order zig-zag functions (Cho & Kim, 2001) were considered. Computational cost was reduced by implementation of this improved theory. The new theory retained the precision of conventional layer-wise theories. The proposed technique also attained the features of variable kinetic theories. Piecewise continuous power series expansions of arbitrary order were carried out on the cross sectional area of the entire layout to achieve those above mentioned features. Laminated and sandwich beams of small length to depth proportionate value were considered to develop various other mathematical from this theory. The new approach has two major advantages. Stress distribution and displacement over the cross-sectional area of the beam is even and accurate. The natural frequencies are also as precise and reliable as that of conventional layer-wise theories.

Vibration characteristics of rotating composite shafts was analysed by Arab et al., (2017a). Equivalent Single Layer Theory (ESLT) was used in that study. The study incorporated rotary inertia, gyroscopic and shear deformation effects so as to examine the dynamic effects of the composite shaft. . The results demonstrated that shear-normal coupling, stacking sequences and fiber orientations attained good effect on the dynamic characteristics composite shafts. Arab et al., (2017b) proposed a multi-layer finite element for rotating laminated shaft. The theory employed the mechanism of layerwise and shaft theories. The theory considered the various characteristics of rotating composite shafts like, critical speeds, natural frequencies and dynamic behaviour. In the dynamic analysis of rotating laminated shaft, the proposed Layerwise Shaft Theory was found to be relevant and applicable Euler-Bernoulli's shaft finite element formulation was proposed by Arab et al., (2018) by applying Equivalent Single layer Theory (ESLT). The proposed formulation too focused on the dynamic analysis of rotating shaft with laminations. The results of this method demonstrated that well in terms of instability threshold and natural frequencies when compared to SHBT and EMBT.

Timoshenko Beam Theory (TBT)

The application of finite elements for the simulation of rotors models which is considered more important in last few years. Nelson, (1980) proposed a model which uses the Timoshenko beam theory for developing shape functions including transverse shear effects. The model consists of the effects of rotational and translational inertia, axial load and shear deformation. A Timoshenko beam finite element model was suggested by Lien-Wen & Der-Ming, (1991). The model focused on the natural rotating speeds of a composite shaft while varying the conditions and thinness ratios. The study revealed the relationship between natural rotors' speeds and conditions of rotating shafts. Rotary inertia and shear deformation also affected the speed of natural rotors. The study also established the superiority of finite element models in producing fine convergence and meticulous rotating shaft systems. The results produce more accurate finite

element model for rotating the dynamic shaft.

Eringen's nonlocal elasticity theory and TBT were employed by Wang et al., (2006) in the examination of elastic buckling properties of micro and nano tubes. Another new model to evaluate the static and dynamic behaviors of the beams which are functionally graded were suggested by Li, (2008). The model considered rotary inertia and shear deformation of the graded beams. This study deduced Euler–Bernoulli, and Rayleigh beam theories through procedural analysis of Timoshenko beam theory. A theory for laminated composite beams was proposed by Tessler et al., (2009) by considering the kinematics properties of the Timoshenko Beam Theory. This provides a relatively accurate estimation of structural response, low-cost, and high-performance aerospace structures.

Rao & Roy, (2016) proposed a dynamic analysis model for functionally graded (FG) rotor shaft system. This theory utilized power-law gradation for material modelling. Timoshenko beam theory (TMBT) was employed for the finite element modelling of FG rotor shaft. FG shaft model is confirmed that the good critical speed by considering rotary inertia, strain, internal damping and shear deformation. An active magnetic bearing (AMB) aid rotors that use electromagnetic force (EMF) instead of mechanical forces. Hence, exact prediction of the coefficient of AMB forces is imperative as rotor dynamic analysis requires system stability, vibrations modes, and critical speeds. A theory to predict the AMB damping coefficients and the stiffness was proposed by Xu et al., (2017). Rotor unbalance characteristics were considered for formulation of the theory. The theory used Timoshenko beam theory to generate the proposed finite element rotor model. An identification procedure was developed through utilization of finite element model. The proposed model exhibited higher efficiency rate than other conventional beam theory techniques.

Euler–Bernoulli Beam Theory

Bauchau et al., (2009) proposed Euler-Bernoulli beam theory for constructing aeronautical composite materials. Zhu & Chung, (2015) presented a new nonlinear coupling effect of the axial and lateral displacements to deploy beam with spin. The new nonlinear model was proposed through assimilation of the Euler-Bernoulli beam theory and the von Karman theory. The study analysed the characteristics of linear and nonlinear models. The implications of parameters of both types of models were also examined. The study found remarkable differences between the working procedure and behaviour of linear and nonlinear models. Fluctuation and alteration in the beat phenomenon due to intrusion of two similar natured frequencies of the spinning beam was observed at the end of the study.

A new theory on the bending of Bernoulli-Euler beam was suggested by Park &Gao, (2006). The study concentrated on enhanced couple stress theory to formulate the revamped model. For this, a differential formulation using minimum total potential energy was applied using internal material length. Euler–Bernoulli beam theory was studied and applied by Alshorbagy et al. (2011) to analyse the transversally functionally graded beams. Free vibration response of the selected beams were examined with the help of finite element. In different studies, Shahba et al., (2011a) &Shahba et al., (2011b) analysed axially functionally graded (AFG) beams. The studies focused on stability and vibration of AFG beams. Euler–Bernoulli and Timoshenko beams were taken as reference beams for these studies. A finite element method and various numerical methods were employed in these researches. . Vibration analysis for non-uniform and non-homogenous micro-beams was carried out by Akgöz&Civalek, (2013). The study assessed and incorporated the threshold of micro-beam. The material characteristics and the cross-sectional properties of the selected micro-beam were dissimilar in nature. It was found that, there are significant impacts of taper ratios and material properties on natural frequencies in case of AFG tapered micro beams. Strain gradient theory was taken as reference by Kahrobaiyan et al., (2011) to analyse a

size-dependent functionally graded (FG) Euler–Bernoulli beam theory.

A new model was suggested by Chung, (2016) to install spinning beam. The study also analysed the properties and activities of the spinning beam. The model came into effect through the implementation of Rayleigh and Bernoulli beam theories. Inertial references and rotary inertia effects of spinning beam were examined under this study. The study also evaluated the beat phenomena related to spinning Rayleigh beam by positioning it in an inertial reference frame. The results provide more precise and reliable outcomes for dynamic behaviors when the spinning beam is utilized. Oke&Khulief, (2016) conducted a study on the finite element. The study employed B-spline wavelets on the interval for research conduction. The objective of the study was to realize free vibration properties of composite pipes. Wavelet space was initially used to design finite element for the study. Then that element was converted into physical space. Wavelet functions and B-spline scaling were applied to obtain the quotient of stiffness and mass of composite element. In this, the design is considered Euler-Bernoulli beam theories which utilize significantly fewer elements compared to conventional techniques. Phadatare et al., (2017) presented the evaluation of nonlinear operations with the aim of the divergences and chaotic performance of an elastically made flexible rotor-bearing system. The study contemplated ground motion with harmonic characteristics. Also, the higher-order Euler– Bernoulli deformation theory was implemented that controls the nonlinear dynamic characteristics of the rotating system.

Shear Deformation Beam Theory (SDBT)

A model analysing the static and dynamic characteristics of narrow edged composite beams was suggested by Suresh and Nagaraj, (1996). Open and close sectioned HSDT was employed for development of the model. The results are confirmed by using analytical comparison and the static deflections of composite beams. Another model for representation of displacement structures was proposed by Marur and Kant, (1996). Free vibration of composite beams was analysed in the study for representation of three higher order frameworks. It was found that models using higher order theories obtained lower frequencies with respect to other existing theories. Buckling and free vibration attributes of stepped composite beams with laminations was examined by Song and Waas, (1997) and Matsunaga, (2001). Both the studies applied HSDT. The obtained outcomes of both the studies were analysed with respect to first-order deformation beam theory. The comparison did not reveal any significant advantage of broad range of beam geometry. Two FEM and two higher order techniques were suggested by Subramanian, (2006) for laminated beams. Symmetrically laminated beams and dissimilar thickness ratios were considered in that study

Chang et al., (2004) proposed a general spinning composite shaft model which was comprised of discrete isotropic rigid disks bearings. Shear deformable beam theory of first order was employed to figure out the strain energy level of the shaft. Three-dimensional constitutive relations of material was also considered for this purpose. Rotating coordinate systems of the shaft was used in this study to obtain the kinetic energy. The energy was obtained through consideration of shaft cross-sectional area. To formulate the required equations for this study, Hamilton’s extended theorem was used. The resulting model incorporated with gyroscopic effects, transverse shear deformation rotary inertia, and coupling effects. The results show that the proposed beam theory provides better transient responses of a particular composite shaft. Trigonometric deformation theory was employed by Jun et al., (2014) to examine laminated shallow curved beams for the analysis of vibration patterns. . In the study, Wittrick-Williams algorithm was used to assess the mode shapes and natural frequencies of the laminated beams with shallow curves. Dynamic stiffness characteristic was utilized for that assessment. . Higher-order shear deformation plate theories were suggested by Yahia et al., (2015) using

functionally graded plates to assess the wave propagation pattern. Lesser number of undefined variables and equations were used in the proposed plate theory as compared to existing conventional first-order shear deformation theories. By solving the eigenvalue problem, the critical dispersion relation of the functionally graded plate was obtained.

Vibration behaviour of functionally graded materials was the foundation for the higher order beam theories of Thai &Vo, (2012) and Sahmani, & Ansari, (2013). . Filippi& Carrera, (2015) proposed a Carrera Unified Formulation (CUF) which was developed through integration of unsteady aerodynamic theories and higher order beam theories. The model analysed aeroelasticproperties of rotating and fixed wings. Multi-sectional and rectangular contour of composite and isotropic materials were analysed for this research model. Impacts of lamination structures, aerodynamic theories, and the order of structural models on the flutter conditions of the beam were also analysed through this study. . The simulation results of the study demonstrated that the CUF provides the excellent performance of the dynamics and aeroelasticity of rotors in terms of accuracy, flutter instability. Shabanlou et al., (2017), Simsek& Reddy, (2013) and Ebrahimi&Barati, (2016) investigated the vibration assessment of FG spinning beam which is applied to thermal environment with the help of higher order SDBT. Also, a displacement field of hollow circular cylindrical spinning beam was developed which maintained the shear stress to be free at both outer and inner surface of the beam.

Comparison of Different Beam Theory Techniques

The comparative analysis of different beam theory techniques for rotating laminated shafts is shown in Table I. This comparison table provides critical speed (in rpm) of the each beam theory technique.

Table 1: Comparison of Beam Theory Techniques

S. No	Paper/Author Name	Beam theory Techniques used	Critical Speed (in rpm)
1.	Analysing the composite shaft rotor with the help of layer wise theory. (Singh & Gupta, 1996)	EMBT	5746
		LBT	5620
2.	The effect of coupling mechanisms and stacking sequence on the natural frequencies of composite shafts (Gubran& Gupta, 2005)	Modified EMBT	5552
3.	Analysis of the dynamics of rotating composite shafts (Sino et al., 2008)	SHBT	5435
4.	Analysis of the dynamics of rotating composite shafts with the help of layerwise theory (Arab et a., 2017b)	LBT	5769
5.	Identifying parameters of active magnetic bearings from the rotor unbalance responses. (Xu et al., 2017)	TMBT	6000
6.	Assessment of nonlinear responses and bifurcating the rotor bearing system which is mounted on a platform that moves. (Phadatare et al., 2017)	Euler-Bernoulli beam theory	2865

CONCLUSIONS

This paper discussed the mechanism of rotating laminating shafts using various beam theory techniques. The beam theory techniques used in this paper include, EMBT, SHBT, Modified EMBT, LBT, TMBT, Euler-Bernoulli beam theory and SDBT. All techniques have its own pros and cons in terms of critical speed, accuracy, natural frequencies and stiffness. In this, modified EMBT offers more efficient performance of tubular composite shafts for dynamic rotor analysis than the EMBT. From this review, the beam theory techniques such as LBT, TMBT and SDBT techniques provide more accurate performance with higher stiffness. In Future, the beam theory techniques can be improved to enhance the dynamic performance of rotating laminated shafts.

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